Frequency-Domain Shuttle-Pulse Measurement of Losses and Mode Conversion

PAOLO BERNARDI, FRANCO BERTOLANI, AND VITTORIO RIZZOLI

Abstract—**A new method for measuring losses and mode conversion at microwave frequencies is presented. The method consists of producing the impulse response of single- or double-cavity resonant circuits in the frequency domain. To obtain this response, the input reflection coefficient of the circuit to be measured is first changed into a function of time by a swept-frequency technique, and then Fourier-transformed by means of a spectrum analyzer system. High generality of application and repeatability as well as measurement simplicity and definition are the main features of the new method. The examples of application presented include measurement of microstrip-line attenuation constants up to X-band and characterization of mode conversion coefficients in overmoded circular waveguide at millimeter-wave frequencies.**

I. INTRODUCTION

THE PURPOSE of this paper is to present a new method for loss and mode-conversion measurement at microwave frequencies. This method has proven to be very effective, since it is simply implemented and gives well defined and repeatable results together with wide application, ranging from microstrip attenuation measurement to evaluation of losses and mode-coupling coefficients in TE_{01} circular waveguides. Several methods are available to date for performing such measurements. Probably the most popular are the resonance method (e.g., [1], [2]) and the shuttle-pulse method (e.g., [3], [4]). In both cases the physical system to be characterized is a cavity which is loosely coupled to the outside world.

In the resonance case, the cavity is excited by a swept RF generator and the reflected signal is detected and displayed on a CRT. Thus the quantity of concern is the squared magnitude of the reflection coefficient. When the loss-to-coupling ratio is properly adjusted, a resonant dip is observed. From the measured value of the resonant frequency and the shape of the dip, the loaded cavity Q as well as other quantities of interest can be derived. However, great difficulty may be encountered while determining the Q factor. Actually, when Q values are high (which is usually the case at high frequencies), estimating the very narrow 3-dB bandwidth with reasonable accuracy may become a very critical job unless very sophisticated test equipment is available [5]. Moreover, due to parasitic effects the dip shape may be slightly irregular, thus making it difficult to establish the reference value. On the other hand, this method can be applied to almost any type of cavity with no regard to cavity length.

In the shuttle-pulse case, the cavity is excited by a pulsed RF generator, and the reflected signal is displayed and observed in the time domain. In particular, when the cavity consists of a short-circuited transmission-line section, the response is made of a train of pulses with an exponential envelope. From this envelope the overall cavity losses can be easily inferred, which is equivalent to finding the loaded Q. Thus the result is essentially the same as obtained from the previous method. In this case, relatively long (a few meters) cavity lengths are required, and arranging the setup is usually a very complicated and lengthy job. On the other hand, determining the Q factor is easy, and the results obtained are well defined and repeatable. Moreover, measurement becomes easier as Q values increase.

The new method has been devised in such a way as to retain the advantages of each of the previous ones, without suffering from their inconveniences. The underlying principle is as follows. For a given cavity, made of a short-circuited transmission-line section, the shuttle-pulse response described above ideally represents the impulse response, while the reflection coefficient $\rho(j\omega)$ is the frequency response. Thus a Fourier-transform relationship exists between these quantities. As will be shown in the paper, it is possible to produce the shuttle-pulse response, yielding more accurate readings, without making use of a shuttle-pulse setup, but by just Fourier-transforming the response of a resonance setup, which is more easily arranged. In practice, this transformation will be performed by a spectrum analyzer system (SA). Since commercially available SA's usually include a built-in logarithmic amplifier, the exponential envelope can be easily reduced to a linear one (see Fig. 2). Thus the measurement is simply performed by estimating the slope of a straight line on a CRT display, which is clearly a very straightforward and reliable operation.

In addition to the single-cavity configuration considered so far, the method can also be applied to a double-cavity scheme. In this case, as will be shown in

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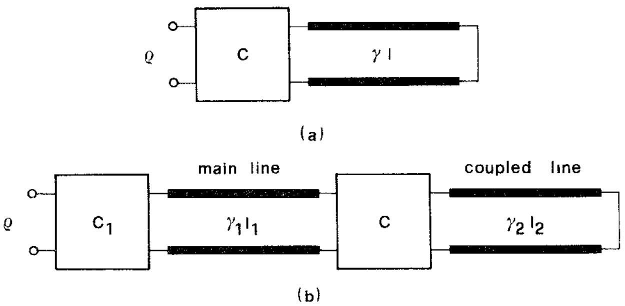

Fig. 1. Equivalent circuit of single- and double-cavity resonators.

Section IV, the envelope of the response displayed by the SA is an exponential modulated by a sinusoid (see Fig. 8), and the measurement is reduced to evaluating the distance between two consecutive zeros of the latter. Note that this is again similar to the result obtained from time-domain measurement [4], thus further confirming the idea that we are actually performing frequency-domain shuttle-pulse measurements.

II. Mathematical Description of the Single-Cavity Case

In this section the measurement procedure for the single-cavity case will be analyzed. The basic circuit configuration to be considered is shown in Fig. 1(a). The network consists of a short-circuited transmission-line section, having propagation constant $\gamma = \alpha + j\beta$ and length l, and a symmetrical lossless two-port C providing small coupling between its input and output ports. It is well known that this network can be used as an equivalent circuit for almost all practical situations of coupling to a resonant cavity. Without loss of generality, we will assume that the scattering matrix of C may be denoted by

$$S = \begin{bmatrix} -r & jt \\ jt & -r \end{bmatrix} \tag{1}$$

where r and t are real numbers and $r = \sqrt{1-t^2} > 0$. If we let $r = e^{-\delta}$, δ represents the coupling loss of the conventional shuttle-pulse test set. From Fig. 1(a), the following expression for the input reflection coefficient is obtained:

$$\rho(j\omega) = \frac{e^{-2\alpha l}e^{-j2\beta l} - e^{-\delta}}{1 - e^{-(\delta+2\alpha l)}e^{-j2\beta l}}. \tag{2}$$

Equation (2) shows that $|\rho|$ exhibits a resonant dip in the vicinity of every frequency such that $\beta l = n\pi$, n integer. In particular, $|\rho| = 0$ at these frequencies if $\delta = 2\alpha l$ (critical coupling).

When the cavity is fed by a swept generator, the reflected signal is amplitude and frequency modulated. Since the useful information is carried by the amplitude modulation law, which is inherently low frequency, the RF signal must be detected. If this is accomplished by a detector diode, a low-frequency signal proportional to $|\rho|^2$ will be available. If we let $\phi = 2\beta l$, from (2) the squared magnitude of ρ appears to be a periodic function of ϕ. Its Fourier-series expansion is given by

$$|\rho|^2 = A + M \sum_{k=1}^{\infty} e^{-k\varepsilon} \cos k\phi \tag{3}$$

where A and M are constants and $\varepsilon = \delta + 2\alpha l$ is the total loss of the cavity. In the vicinity of a resonant frequency f_0, ϕ can be approximately represented by

$$\phi \simeq 2n\pi + 4\pi\tau_0(f - f_0) \tag{4}$$

where τ_0 is the group delay at f_0. Usually one can assume $\tau_0 = \frac{1}{2}\Delta f$, where Δf is the frequency distance between two consecutive resonances. When the reflection coefficient is observed by a swept-frequency technique, in (4) we have

$$f - f_0 = \frac{B_S}{T} t \qquad -\frac{T}{2} \leq t \leq \frac{T}{2} \tag{5}$$

where B_S and T are the sweep width and period, respectively, and t is time.

Thus (3) becomes a periodic function of time having period T. If in particular we let $B_S = \Delta f$, (3) directly represents the Fourier series for the input signal. It is thus evident that the spectrum of this signal has an exponential envelope expressed by

$$E(f) = Me^{-\varepsilon T f} \tag{6}$$

so that $\varepsilon = \delta + 2\alpha l$ represents the decay constant of the envelope expressed in nepers per spectral line (Tf is the number of spectral lines or echoes comprised between 0 and f).

In principle, experimental methods are available allowing the loss and coupling contribution to the overall cavity losses to be separated. In practice, these methods are easily implemented if information on the phase of the reflection coefficient at resonance is readily available. This happens, for instance, when measurements are performed in the band up to 18 GHz, making use of a network analyzer. Otherwise, their practical application is cumbersome [2], so that the measurement is best performed by achieving the conditions for critical coupling. In turn, this can be obtained in most practical cases by changing coupling, rather than losses [6], which means that the single-cavity situation considered so far is more suitable for loss than for coupling measurements. When coupling is primarily of concern, a double-cavity configuration such as described in Section IV is often preferred.

III. Experimental Results: Single-Cavity Case

In order to check the validity of the theoretical discussion and the accuracy of the new method, a set of measurements was first performed on a microstrip transmission line in the frequency range from 2 to 12 GHz. The reasons for this choice are as follows. First, in the above band the loaded Q of the microstrip line is low enough (ranging approximately from 70 to 130 in the case considered here), so that the conventional resonance method can be successfully applied. Thus a reasonably well defined set of reference results can be made available for comparison. Second, if the measurement procedure described in [6] is followed, obtaining the critical

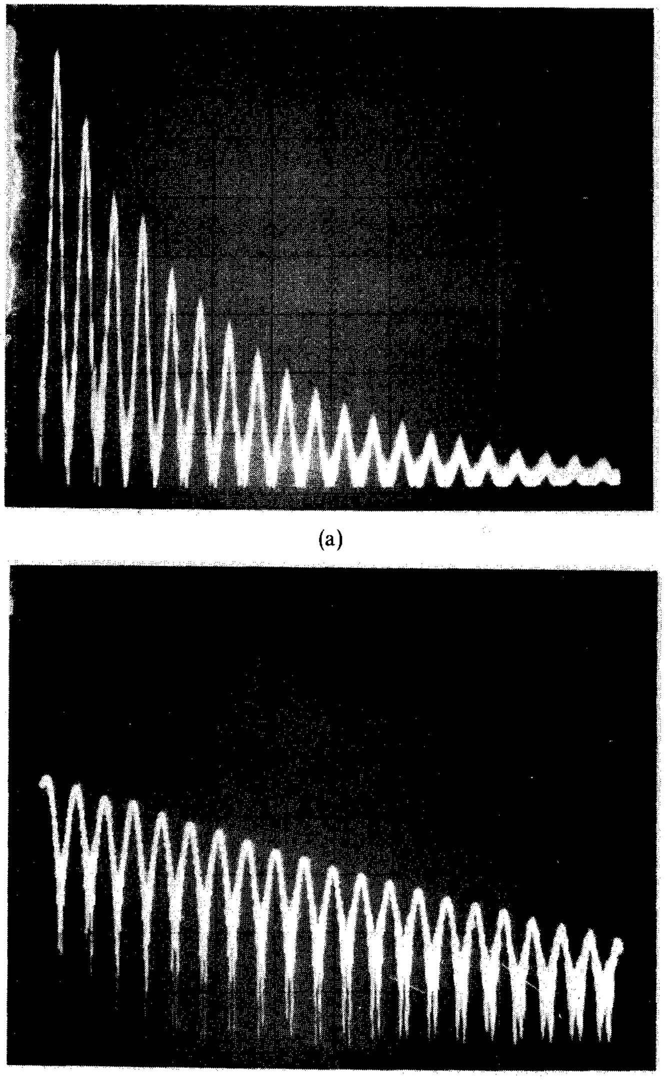

Fig. 2. Frequency-domain shuttle-pulse response of a single-cavity resonator. (a) Linear y axis. (b) Logarithmic y axis.

coupling is an easy and straightforward job, so that no difficulty is encountered in separating loss from coupling effects. Fig. 2 shows the shuttle-pulse response of a 5-cm long section of microstrip line as observed on the CRT display of the SA with linear and logarithmic y axis. The exponential envelope of the spectrum is clearly put into evidence by these pictures. The microstrip was 0.5 mm wide and 3 μm thick and was built on 0.635-mm alumina substrate.

The measured values of the microstrip attenuation constant are plotted against frequency in Fig. 3 (dots), where they are compared with the results obtained from the resonance method (asterisks). The agreement between the two sets of data is excellent, which is a clear proof of the validity of the new measurement procedure.

At higher frequencies, due to the very high Q values usually encountered (up to several thousands), the results obtained from the conventional resonance method are far less satisfactory. On the other hand, as the Q factor is increased, the number of spectral lines that can be observed becomes larger and larger, so that the new method yields

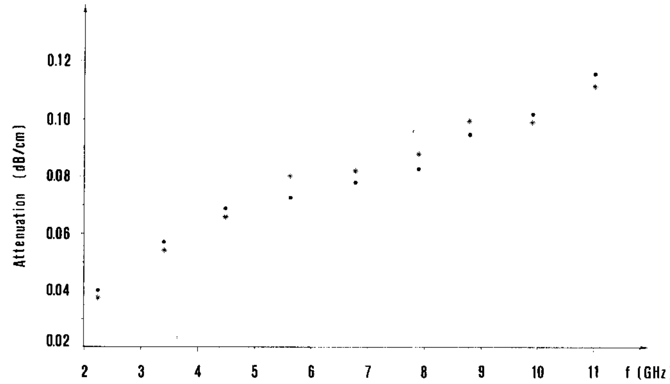

Fig. 3. Measured attenuation constant of a microstrip line obtained by the resonant method (asterisks) and the present method (dots).

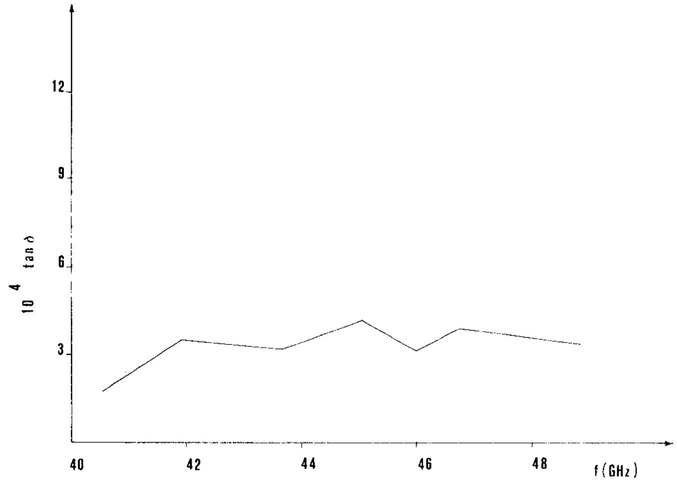

Fig. 4. Measured loss factor of polyethylene sample.

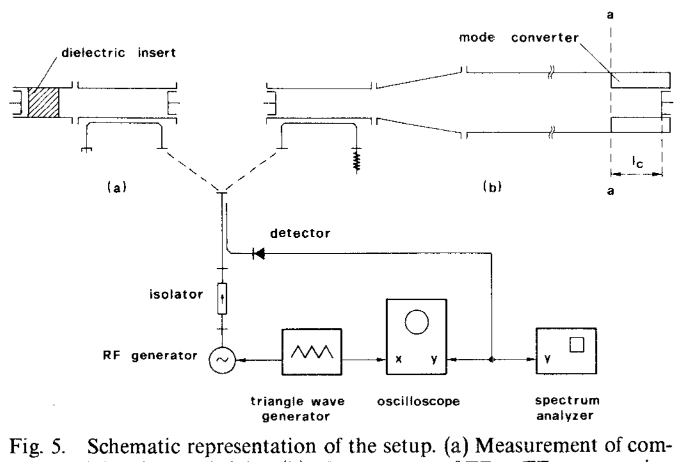

Fig. 5. Schematic representation of the setup. (a) Measurement of complex dielectric permittivity. (b) Measurement of TE_{01}–TE_{02} conversion in circular waveguide.

increasingly sharp and well-defined readings and is to be preferred. As a typical example, the measured loss factor of a low-loss dielectric material (polyethylene) in the frequency range from 40 to 50 GHz is plotted in Fig. 4. The circuit configuration for this measurement is shown in Fig. 5(a). The resonant cavity is obtained from a short-circuited circular waveguide propagating the TE_{01} mode. The coupling mechanism is realized by a TE_{10} to TE_{01} directional coupler with two short-circuited ports. The resonant frequency and cavity losses are measured twice, with and without the dielectric insert. The change in the Q factor can be easily related to the dielectric loss tangent, once the relative permittivity has been found from the resonance frequency shift. The small dispersion of the experimental points around the average gives a clear demonstration of the sensitivity and well defined behavior of the present method. In fact, it should be remembered that these data are obtained by difference between the measured losses of two cavities having very high Q factors of similar magnitude.

IV. Mathematical Description of the Double-Cavity Case

As was anticipated in Section II, when a coupling coefficient has to be found, a double-cavity configuration can be used to perform the measurement, as an alternative to the single-cavity scheme described above. In Fig. 1(b) a schematic representation of the basic circuit is shown. In this case, we are faced with two transmission-line sections and two coupling networks, C_1 and C. The block C represents the coupling mechanism to be measured, which is modeled as a symmetrical lossless two port whose scattering matrix is again given by (1). Thus t is now the unknown parameter to be determined. The block C_1 is a known coupling network providing small coupling between the main line and the measuring equipment. Its scattering matrix will be assumed of the form

$$S_1 = \begin{bmatrix} 0 & jt_1 \\ jt_1 & -r_1 \end{bmatrix}. \tag{7}$$

The input port of C_1 is assumed to be matched in order to simplify the analytical developments. This situation can be easily realized in practice, for instance by means of a directional coupler with one port connected to a matched load and one short-circuited port (see Fig. 5(b)).

As in Section II, we introduce the coupling losses δ_1 and δ defined by $r_1 = e^{-\delta_1}$, $r = e^{-\delta}$. From Fig. 1(b) the input reflection coefficient can be expressed as

$$\rho(j\omega) = \frac{-t_1^2 e^{-2\alpha_1 l_1 - j\phi_1}[-e^{-\delta} + e^{-\varepsilon_2 - j(\phi_1 - \Delta\phi)}]}{1 - 2e^{-\delta - \Sigma\varepsilon/2 - j(\phi_1 - \Delta\phi/2)} \cosh\left(\dfrac{\Delta\varepsilon + j\Delta\phi}{2}\right) + e^{-\Sigma\varepsilon - j(2\phi_1 - \Delta\phi)}} \tag{8}$$

where

$$\begin{aligned} \varepsilon_1 &= \delta_1 + 2\alpha_1 l_1, \\ \varepsilon_2 &= 2\alpha_2 l_2, \\ \Sigma\varepsilon &= \varepsilon_1 + \varepsilon_2, \\ \Delta\varepsilon &= \varepsilon_1 - \varepsilon_2, \\ \phi_1 &= 2\beta_1 l_1, \\ \phi_2 &= 2\beta_2 l_2, \\ \Delta\phi &= \phi_1 - \phi_2. \end{aligned} \tag{9}$$

In order to perform the measurement at a given frequency f_0, the lengths of the transmission lines are adjusted in such a way that both resonate at f_0, and their group delays are approximately equal at this same frequency (that is, $\tau_1 = \frac{1}{2}\Delta f_1 \simeq \tau_2 = \frac{1}{2}\Delta f_2$). These conditions are easily met in the commonly encountered cases of identical cavities or coupling between modes well above cutoff in overmoded waveguides. Thus at resonance we have $\phi_1 = 2n_1\pi$, $\phi_2 = 2n_2\pi$ with n_1 and n_2 integers. Moreover, if we approximate the electrical lengths in the vicinity of f_0 in the same way as in Section II (see (4)), then we have $\Delta\phi \simeq 2(n_1 - n_2)\pi$ in the frequency band of interest. In practice, the above conditions can be easily checked by observing the positions of the resonant dips on a conventional oscilloscope.

By letting $\phi_1 = \phi$, (8) becomes

$$\rho(j\omega) = \frac{-t_1^2 e^{-2\alpha_1 l_1 - j\phi}(e^{-\varepsilon_2 - j\phi} - e^{-\delta})}{1 - 2\cos\psi e^{-\Sigma\varepsilon/2 - j\phi} + e^{-\Sigma\varepsilon - j2\phi}} \tag{10}$$

with

$$e^{-\delta}\cosh\frac{\Delta\varepsilon}{2} = \cos\psi. \tag{11}$$

At this stage, the same procedure as in the single-cavity case can be followed. The Fourier-series representation of $|\rho|^2$ is now

$$|\rho|^2 = M\left[\frac{|\sin\theta|}{2} + \sum_{K=1}^{\infty} e^{-K(\Sigma\varepsilon/2)} \sin(K\psi - \theta)\cos K\phi\right] \tag{12}$$

where M is a constant and θ is defined by

$$\tan\theta = -\frac{\sin\psi}{\sinh\dfrac{\Sigma\varepsilon}{2}} \cdot \frac{\cosh(\delta - \varepsilon_2)\cosh\dfrac{\Sigma\varepsilon}{2} - \cos\psi}{\cosh(\delta - \varepsilon_2)\cos\psi - \cosh\dfrac{\Sigma\varepsilon}{2}}. \tag{13}$$

Equation (12) is the analogue of (3) for the double-cavity configuration. As in the previous case (see Section II), when the reflection coefficient is observed by a swept-frequency technique, (12) is changed into a periodic function of time. If the sweep bandwidth is $B_S = \Delta f_1 = \Delta f_2$ and is centered on f_0, then (12) by means of (5) represents the Fourier expansion of the input signal to the SA. Again, the spectrum consists of an infinite set of discrete lines; the envelope is expressed by

$$E(f) = Me^{-(\Sigma\varepsilon/2)Tf}\sin(\psi Tf - \theta) \tag{14}$$

where T is the sweep period.

From (14) it is evident that now zeros appear in the envelope as far as $\cos\psi < 1$, that is, ψ is real. These are given by

$$Tf_m = \frac{\theta}{\psi} + m\frac{\pi}{\psi}, \qquad m = 0, 1, 2, \cdots \tag{15}$$

and are clearly spaced by π/ψ.

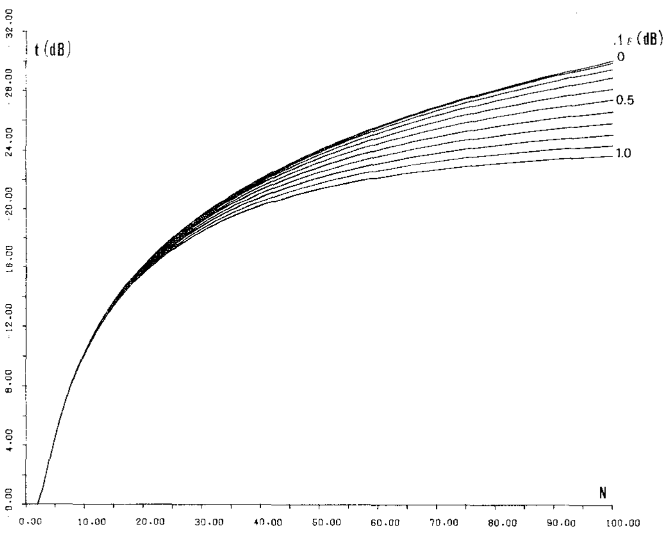

Fig. 6. Coupling coefficient t versus number of spectral lines N with $\Delta\varepsilon$ as a parameter.

The most general way of performing the measurement is to experimentally evaluate the frequency distance between two consecutive zeros, which directly yields π/ψ. In practice, since the product Tf represents the number of spectral lines comprised between 0 and f, we are reduced to counting the number N of lines embedded between two consecutive zeros. In this way we are able to perform an absolute measurement, that is, the errors on T and f arising from inaccurate calibration of the instruments do not affect the result. Thus $N = \pi/\psi$, and from (11)

$$t^2 = 1 - \frac{\cos^2 \dfrac{\pi}{N}}{\cosh^2 \dfrac{\Delta\varepsilon}{2}}. \tag{16}$$

From (16) the unknown coupling coefficient t can be obtained provided that $\Delta\varepsilon$ be known. Note that the minimum value of coupling that can be theoretically measured by this technique for a given $\Delta\varepsilon$ is $t_{\min} = \tanh(\Delta\varepsilon/2)$ (which is obtained for $N = \infty$ in (16) or $\cos\psi = 1$ in (11)). The relationship between t (in dB) and N is plotted in Fig. 6 with $\Delta\varepsilon_{\text{dB}}$ as a parameter ($\Delta\varepsilon_{\text{dB}} = (20/(\ln 10)\,\Delta\varepsilon)$.

To find $\Delta\varepsilon$, first note that $\Sigma\varepsilon/2$ can be directly read on the CRT display of the SA, since it represents the decay constant of the spectrum by (14). On the other hand, a very good guess to the value of ε_1 can be obtained by changing the length of the coupled line so that $\phi_2 = (2n_2 - 1)\pi$. In this way, we are reduced to a situation similar to that depicted in Section II, that is, to a single-cavity behavior, as can be easily inferred from (8). All we observe now is an exponentially decaying spectrum whose decay constant is given by $\bar{\varepsilon} = \varepsilon_1 + (t^2/4)\varepsilon_2$, which is practically indistinguishable from ε_1 in all cases of technical interest. Thus

$$\Delta\varepsilon \simeq 2\bar{\varepsilon} - \Sigma\varepsilon. \tag{17}$$

Further, note from Fig. 6 that the coupling coefficient for fixed N is a slowly varying function of $\Delta\varepsilon$, so that a slight

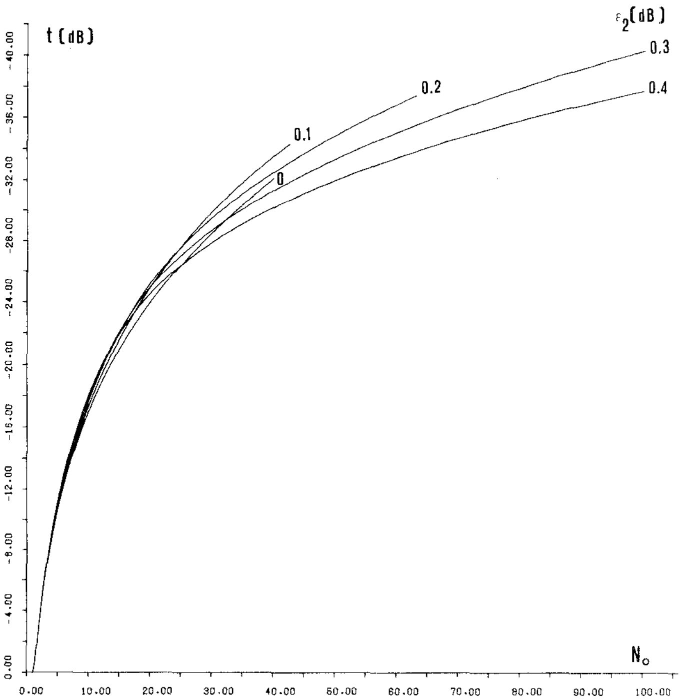

Fig. 7. Coupling coefficient t versus number of spectral lines N_0 with $\varepsilon_1 = 0.43$ dB and ε_2 as a parameter.

mistake in evaluating the latter does not affect significantly the final measurement.

When the coupling coefficient to be measured is very low (typically lower than -30 dB) and/or the cavity losses are high, performing the measurement in the way described so far may become impossible because just one zero of the envelope may be observable. If this is the case, the unknown coupling coefficient can still be measured in the following way. From (15) with $m = 0$, the number of spectral lines encountered between dc and the first zero in the envelope is given by

$$N_0 = \frac{\theta}{\psi} \tag{18}$$

where θ and ψ are expressed by (13) and (11). Equation (18) establishes a relationship between N_0 (that can be measured) and the unknown coupling t, with ε_1 and ε_2 as parameters. While ε_1 can be experimentally found as described above, no simple means is now available for finding ε_2. However, a sufficiently accurate value of the coupling coefficient can still be found by a reasonable guess to ε_2, since an exact knowledge of ε_2 is not required. As an example, see Fig. 7, where the relationship (18) is plotted with fixed ε_1 ($\varepsilon_1 = 0.43$ dB) and ε_2 as a parameter.

V. Experimental Results: Double-Cavity Case

To check the validity of the theoretical conclusions of Section IV, a set of measurements was first carried out on a known TE_{01}–TE_{02} mode converter in circular waveguide. The corresponding setup is schematically illustrated in Fig. 5(b).

The circuit under measurement is made of a section of

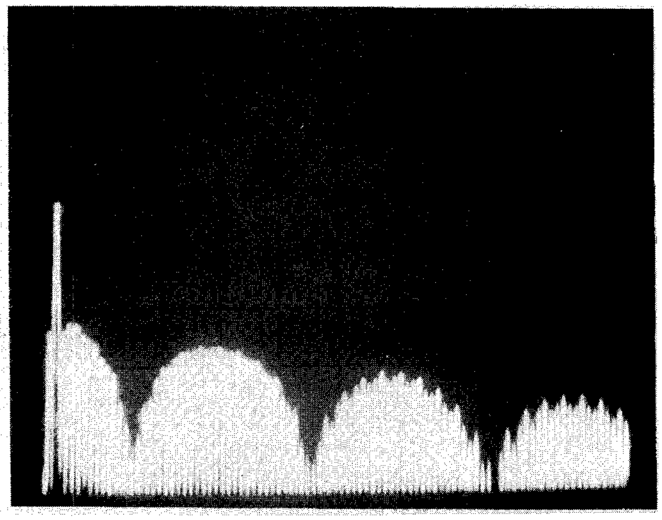

Fig. 8. Frequency-domain shuttle-pulse response of a double-cavity resonator with logarithmic y axis.

51-mm ID circular waveguide with a mode converter inserted at one end. As shown in the figure, the mode converter consists of a variable length l_c of reduced diameter (48 mm) waveguide. The step occurring in section a–a when separately considered is a converter between the TE_{01} and TE_{02} modes of the circular waveguide, corresponding to the main and coupled lines in Fig. 1(b), having a conversion coefficient c_0. Due to the presence of the short-circuited waveguide section after the step, from section a–a an equivalent mode converter having a coupling coefficient

$$t = 2c_0\sqrt{1-c_0^2}\left|\sin 2\pi \frac{l_c}{\lambda_B}\right| \tag{19}$$

is seen, where λ_B is the beat wavelength between the TE_{01} and TE_{02} modes. This equivalent converter corresponds to the unknown coupling network in Fig. 1(b). In the present case, the geometry was chosen in such a way that $20 \log_{10} c_0 = -20$ dB at the frequency of measurement (39.4 GHz), and the equivalent conversion was varied by changing l_c. The input coupling network C_1 was realized by a TE_{10}-rectangular to TE_{01}-circular waveguide directional coupler and a tapered transition providing a TE_{01}–TE_{02} conversion less than -50 dB at 39.4 GHz.

A typical example of the response obtained from this setup as displayed on the CRT of the SA is shown in Fig. 8 in logarithmic scale. In Fig. 9 the measured conversion coefficient t (in dB) is plotted against l_c/λ_B and compared with the exact theoretical values as computed by (19) (solid line). Two sets of measured data are reported in this figure. The asterisks were obtained by observing two consecutive zeros and using (16), while the dots were obtained from the first zero of envelope through (18) and (13). The measured values of $\varepsilon_1,\varepsilon_2$ were, for the present case, $\varepsilon_1 = 0.43$ dB, $\varepsilon_2 = 0.21$ dB independent of l_c. As a consequence, the theoretical sensitivity for this case is $t_{\min} \simeq -38$ dB. This prediction is fully confirmed by the experimental results

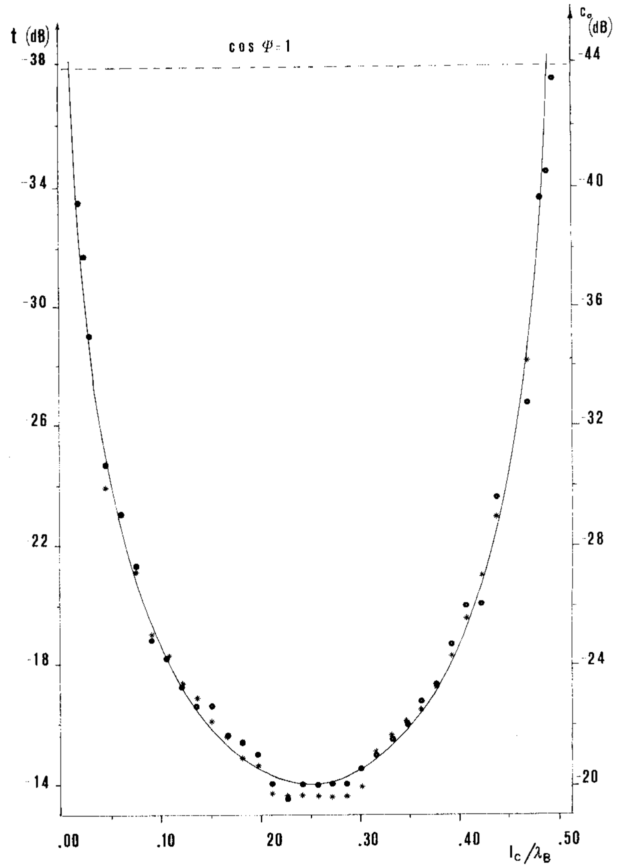

Fig. 9. Measured conversion coefficient versus converter length obtained from first zero (dots) and two consecutive zeros (asterisks). Solid line is theoretical. The horizontal dashed line shows the theoretical limiting value for the present case.

shown in the figure, where a measured value of -37.5 dB is reported.

Making use of (19), the preceding results can be interpreted in a slightly different way. Assume that the unknown to be measured is the intrinsic conversion coefficient of the step c_0, instead of the equivalent conversion coefficient t. Then by making $l_c = \lambda_B/4$ (which is the most favorable condition [4]), from (19) one obtains ($c_0 \ll 1$)$c_0 \simeq t/2$, or, in decibels, c_0 (dB) $\simeq t$ (dB) $- 6$. This means that a step (or, in principle, any other coupling mechanism) having a conversion coefficient as low as -43.5 dB could be measured.

The values of c_0 that one would measure in this way are also reported in Fig. 9. Note that the result in this kind of application is not very sensitive to an error in the position of the short circuit since the sinusoidal term in (19) is stationary at $l_c = \lambda_B/4$.

As the accuracy is concerned, note that an inaccurate evaluation of the number of spectral lines affects the precision of the result less and less as coupling decreases, thanks to the particular shape of the curves reported in Figs. 6 and 7. As a consequence, though a correct evaluation of the number of spectral lines becomes more and more difficult as coupling is decreased, the relative accuracy of the measured results is not degraded as clearly appears from Fig. 9.

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Coupling of Waveguides Through Large Apertures

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Abstract—**A closed form expression for the equivalent network of a narrow transverse slot in the common broadwall between two waveguides is derived in terms of self-reaction and discontinuity in modal voltage. The coupling is expressed in terms of equivalent circuit parameter. A comparison between theoretical results with those obtained from the results of Levinson and Fredberg (for slot length $> 0.4a$, where a is the broad dimension of the waveguide), the results obtained from the formula of Sangster and the experimental results are presented.**

I. Introduction

IT HAS BEEN established by Levy [1] that the synthesis of a well-matched highly directive waveguide coupler reduces to the problem of designing a well-matched filter consisting of a cascade of shunt or series reactances. The application of the method demands an exact knowledge of the even and odd mode equivalent circuit parameters of the aperture in the common wall between two coupled waveguides. Further, the equivalent circuit representation makes possible the rapid determination of the waves emerging at all ports of a coupler using single as well as multiple apertures. In the latter case, it permits inclusion of the effects of reflection in between the apertures. The method of determination of equivalent circuit parameter and coupling therefrom, presented by Levy is limited to the apertures having small dimensions compared with wavelength. Marcuvitz [2] has given an expression for the susceptance presented by a transverse slot in the common broadwall between two waveguides, which is valid for the length of the slot equal to the broad dimension of the guide. Levinson and Fredberg [3] also have presented an analytical method for the determination of the equivalent circuit of apertures in the form of long narrow slots. Evaluation of the equivalent network by this method involves integrals of input admittances of coupled volumes, expressions for which have to be found from the solution of an integral equation [4]. No closed-form expression has been given for the aperture reactance as a function of slot length. For the particular case of two identical standard X-band rectangular waveguides, the curves for the reactance variation presented by Levinson and Fredberg [3] are valid for $2\ell/a > 0.4$.

The problem of electromagnetic coupling has been analyzed by a number of investigators [5]–[7]. The limitations of each of these methods have been discussed by Sangster [8] who applied the variational method for the analysis of coupling through narrow slots in the common wall between two rectangular waveguides. Sangster derived an expression for coupling between two waveguides coupled through slots in the common broadwall by employing an appropriate magnetic dyadic Green's function and using first-order trial function for the electric field distribution in the slot. He presented some computed and experimental results on coupling for a transverse slot in the common broadwall. The evaluation of coupling computed by the authors using Sangster's formula shows a deviation of 3 dB from the maximum coupling obtained from Levy's formula using an equivalent circuit approach, as well as with the experimental results by authors.

In the present paper a closed-form expression for the equivalent network parameter of a transverse slot in the common broadwall of a waveguide is derived in terms of self-reaction [9] and discontinuity in modal voltage [10]. The volume integral appearing in the expression for self-reaction is evaluated by replacing the slot by its equivalent cylindrical

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